Stability of elongated cross-section tokamaks to axisymmetric even poloidal mode number deformations

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In a recent paper, Nakayama, Sato, and Matsuoka [Phys. Fluids **31**, 630 (1988)] suggested that elliptical cross-section tokamaks with aspect ratio $R/a = 3.2$ and with elongation $\kappa = 2.6$ are unstable to a splitting ($m = 2$, $n = 0$) instability for plasma $\beta > 5\%$, and that $\kappa \geqslant 4.0$ plasmas are unstable to a splitting for $\beta \geqslant 1\%$. The magnetohydrodynamic evolution code TSC [J. Comput. Phys. **66**, 481 (1986)] indicates, however, that such plasmas are robustly stable with respect to this splitting. In fact, a $\kappa = 3.7$ plasma with $\beta = 23.0\%$ shows no tendency to split. However, the addition of pinching coils at the waist will cause the plasma to split if the current in these coils exceeds a critical value I_c, which decreases with increasing beta.

I. INTRODUCTION

In a recent paper[1] numerical simulation results were presented, which seem to indicate that when the plasma beta in an elliptical cross-section tokamak exceeds a critical value, it becomes unstable to an ideal pressure-driven instability, which causes the ellipse to split into two separate pieces. This result, if correct, would appear to contradict previous analytical results,[2–4] which say that elliptical cross-section plasmas of all elongations are stable to all even mode number axisymmetric deformations. It would also seemingly contradict results we have obtained over the last several years while performing numerous simulations of the axisymmetric evolution and stability of noncircular cross-section tokamak plasmas using the magnetohydrodynamic (MHD) simulation code TSC.[5] This code has been extensively verified both by performing convergence studies and comparing test problems against analytic solutions, and by detailed comparison of the simulation results with data from several large experiments.[6,7]

II. SPONTANEOUS SPLITTING

To investigate splitting modes in tokamaks, we have utilized the configuration of the proposed TCV tokamak.[8] The vacuum vessel and the external coils of the TCV tokamak are modeled by single grid conductors, as illustrated in the figures and listed in Table I. Additional divertor coils and pinching coils on both sides of the plasma have been introduced to allow the investigation of divertor plasmas and to squeeze the plasma waist in order to produce doublets and induce splitting. The locations and currents in these coils are also listed in Table I.

In order to determine if the plasma spontaneously splits, as suggested by Ref. 1, we produced configuration 1, listed in Table I and shown in Fig. 1, which is a high beta, high elongation reference equilibrium for TCV.[9] This plasma has $I_p = 1.2$ MA, $\beta = 23.6\%$, and an elongation $\kappa = 3.7$. For the initial equilibrium we take the pressure and toroidal field profiles to be of the form $p(\tilde{\psi}) = p_0 \tilde{\psi}^{\alpha_p}$ and $g^2(\tilde{\psi}) = g_0^2 - g_1 \tilde{\psi}^{\alpha_g}$, where $\tilde{\psi}$ is the normalized poloidal flux and $\alpha_p = 1.2$, $\alpha_g = 0.3$. This yields a poloidal beta of $\beta_p \cong 1.0$ and an internal inductance of $l_i/2 = 0.16$. The TSC code evolves the equilibrium into a resistive steady state.[5] During the evolution the plasma density was kept at 10× the Murakami density limit so as to shorten the skin time, allowing steady state to be obtained in a reasonable time. We maintained the beta of this configuration at $\beta = 23.6\%$ by adding an energy source to force the pressure to keep its analytic profile, and evolved the magnetohydrodynamic (MHD) Maxwell and transport equations for over two seconds in real time (several skin times), with no signs of splitting. The only feedback systems present in this calculation were a "perfect" Ohmic heating (OH) system,[6] which supplies a toroidal electric field to the boundary as needed to keep the plasma current constant, and a simple radial feedback system that adjusted the current in coils 7 and 8 to keep the outermost flux surface centered.

We conclude that a tokamak plasma of this elongation will not spontaneously split, even at large beta values well above the ballooning mode stability limit.

TABLE I. Coil locations and currents used in simulations.

	Position		Configuration	
Coil	R(m)	Z(m)	I_1^0(kA)	I_2^0(kA)
1	0.475	± 0.100	-322	-277
2	0.475	± 0.300	-216	-215
3	0.475	± 0.500	-313	-280
4	0.475	± 0.700	-215	-183
5	1.300	± 0.750	-29	-40
6	1.300	± 0.600	-153	-164
7	1.300	± 0.300	-66	-66
8	1.300	± 0.150	-212	-167
9[a]	0.850	± 0.875	0	100
10[a]	0.600	0.000	0	$0 \sim -200$
11[a]	1.150	0.000	0	$0 \sim -200$

[a] These coils are not present in the actual TCV experiment.

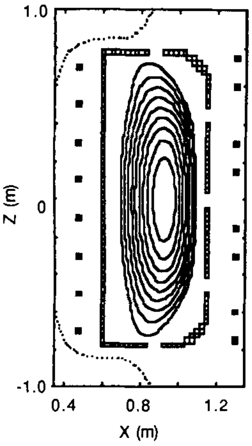

FIG. 1. Flux surfaces of the $\beta = 23.6\%$, $\kappa = 3.7$ equilibrium, which is stable to splitting.

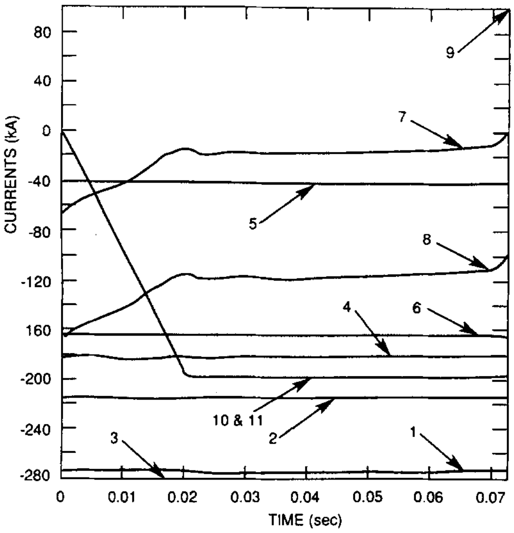

FIG. 3. Time history of the coil currents used to force splitting.

III. FORCED SPLITTING USING PINCHING COILS

In order to study forced splitting of an elongated diverted tokamak, we have introduced a set of "pinching" coils at the waist of the TCV configuration, and have made a study of the critical current in these coils needed to deform the plasma into either a stable doublet configuration with two magnetic axes or to split it into two droplets. The configuration we use in the remainder of the studies is similar to that shown in Fig. 2. This has $I_p = 1.2$ MA, $\beta = 11\%$, and $\kappa = 3.0$. For this case we used $\alpha_p = 1.2$ and $\alpha_g = 1.24$, which results in a poloidal beta of $\beta_p \cong 0.5$ and an internal inductance of $l_i/2 = 0.2$. The placement of the splitting and divertor coils is also illustrated in Fig. 2. (It is listed as configuration 2 in Table I.)

To demonstrate splitting in configuration 2, we ramped the current in the pushing coils 10 and 11 from 0 to -200 kA in 20 msec. The other coil currents were held constant except for coils 7 and 8, which are part of the radial control feedback system, and had their currents adjusted in time to keep the outermost flux surface centered. The "perfect" OH system supplied a boundary toroidal electric field to keep the plasma current constant at $I_p = 1.2$ MA. The time history of these currents is shown in Fig. 3.

The time history of the Z position of the magnetic axis is shown in Fig. 4. At $t = 19$ msec, just about the time the pushing coils reach full power, the plasma transforms into a doublet configuration. This doublet configuration persists, with the separation between the two axes increasing slowly in time, until $t = 70$ msec, at which time the separation increases more rapidly until the doublet splits into two droplets at $t = 72.7$ msec. Three snapshots of the flux surfaces during this evolution are shown in Fig. 5, and in Fig. 6 we show a summary plot illustrating a superposition of the 99% flux surface locations during the evolution.

Next, we repeated the identical case as just described

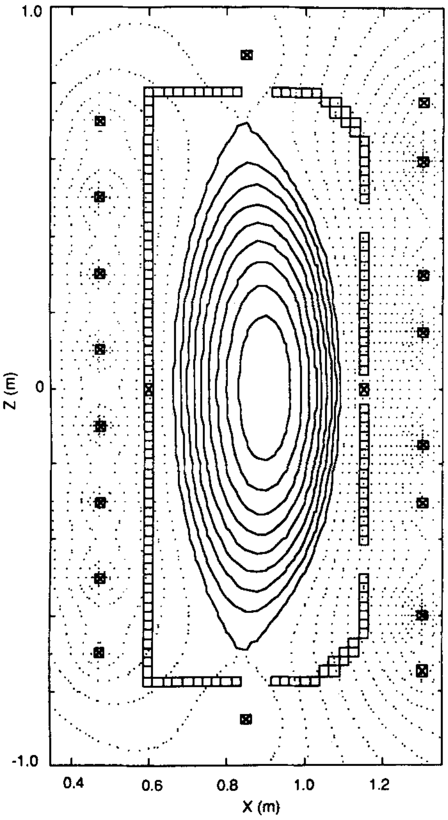

FIG. 2. Flux surfaces of the $\beta = 11\%$, $\kappa = 3.0$ diverted equilibrium showing divertor and pinching coils.

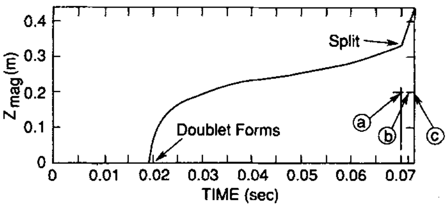

FIG. 4. Time history of the magnetic axis Z coordinate. A doublet forms at $t = 19$ msec and the plasma splits at $t = 70$ msec.

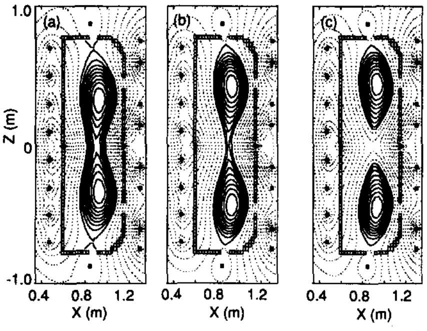

FIG. 5. Magnetic surfaces at the three times marked in Fig. 4 showing tokamak splitting.

(configuration 2), but ramped the pushing coil currents only to − 175 kA instead of − 200 kA and held them at this value. The Z position of the magnetic axis is shown in Fig. 7. It is seen that a stable doublet configuration is formed, with each magnetic axis located at $Z_{\text{mag}} = \pm 0.145$ m. This configuration remained stationary until the calculation ended at $t = 0.8$ sec, longer than all relevant time scales. The magnetic surfaces of the stable doublet configuration at $t = 0.8$ sec are shown in Fig. 8. The TSC simulation that produced this result used a computational grid with cell size $\Delta X = \Delta Z = 0.0125$ m. The convergence of Z_{mag} as the grid size is varied is shown as the indented plot in Fig. 7. Extrapolation to zero grid size yields $Z_{\text{mag}} = \pm 0.130$ m.

To get the plasma to split into droplets for the − 175 kA configuration, we repeated this run, but began increasing the plasma beta by increasing the pressure starting at $t = 0.3$ sec. As shown in Fig. 9, the increased beta causes the plasma to split at $t = 0.43$ sec, $\beta = 13\%$. Another extreme case, shown after evolving for 1.5 sec in Fig. 10, has $\beta = 25.7\%$ and a

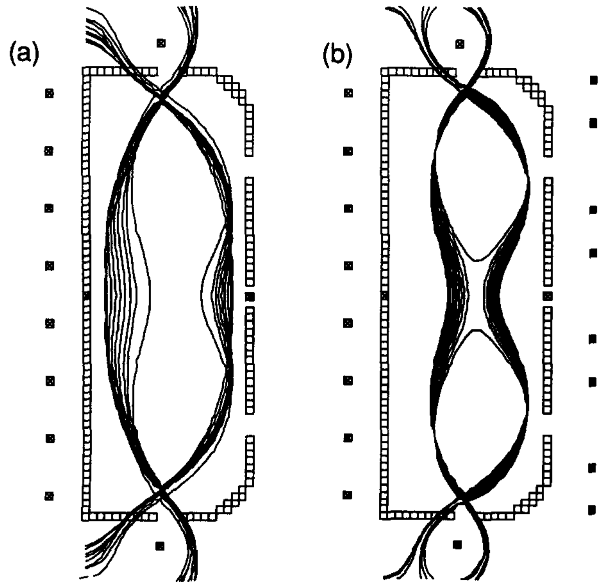

FIG. 6. Summary of the 99% flux surface location for (a) $0 < t < 0.070$ sec and (b) $0.070 < t < 0.072$ sec.

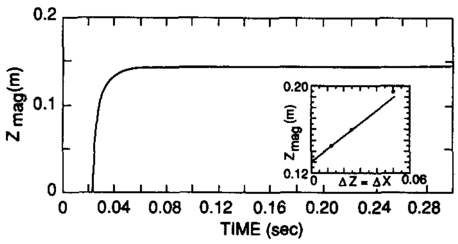

FIG. 7. Magnetic axis Z position versus time for a stable doublet with $I_{\text{pinch}} = -175$ kA. Also, convergence of Z_{mag} with respect to the computational grid size.

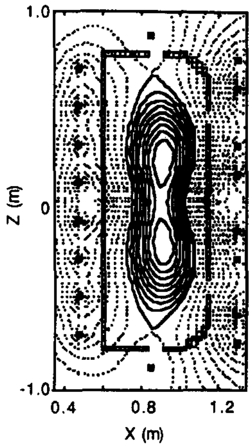

FIG. 8. Magnetic flux surfaces for a stable doublet at $t = 0.8$ sec in Fig. 7.

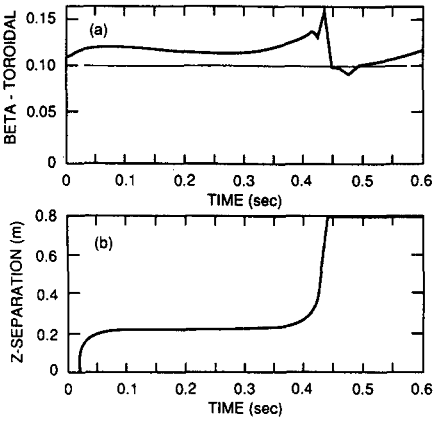

FIG. 9. Time history of the plasma beta and magnetic axis Z position. The plasma splits at $t = 0.43$ sec, $\beta = 13\%$.

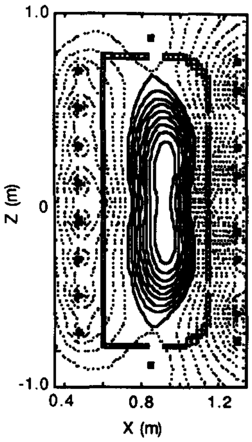

FIG. 10. Flux surfaces of the stable $\beta = 25.7\%$ case with a pushing coil current of − 100 kA.

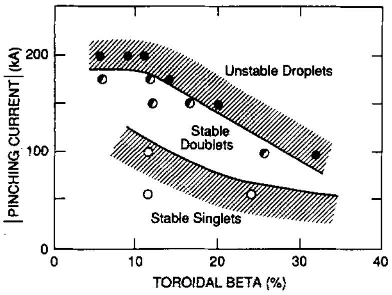

FIG. 11. Critical current in pushing coils as a function of plasma beta.

pushing coil current of — 100 kA. This is again stable, being stationary on all time scales. We have also repeated a simulation with a more peaked pressure profile ($\alpha_p = 2$; $I_{\text{pinch}} = 75$ kA) and have obtained essentially identical results. Thus we conclude that the effect of a specific profile shape plays a minor role for stability.

IV. SUMMARY AND DISCUSSION

In Fig. 11, we show a summary of the runs described in the text, and some additional cases not described. For each beta value considered, up to 36%, there is a critical value of current in the pushing coils that causes the plasma to split. We have *never* seen a tokamak split for any elongation or any beta, unless pinching coils with nonzero current are placed at the waist to force the splitting. The current in these coils necessary to induce splitting does decrease with increasing beta. When no pinching coil currents were applied, we were able to get a stable single magnetic axis equilibrium even with $\kappa = 7$ and $\beta = 70\%$.

Below the critical values for splitting, we found a broad range of stable single magnetic axis configurations, and doublets with two magnetic axes, even at high pressure. Figure 11 also indicates the critical limits where the plasma bifurcates from a singlet to a doublet configuration. Our explanation for bifurcation is that the surrounding coils must be strong enough to produce a saddle point at the original single magnetic axis position. This leads to a reconnection of the innermost flux surfaces and causes the magnetic axis to move off the midplane. After bifurcation the plasma either attains a stable configuration or it splits, depending on the pressure and the pinching coil currents. We conclude that the splitting instabilities are of no concern for presently envisaged advanced tokamaks.

ACKNOWLEDGMENTS

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